

BY HENGJIAN CUI

Write (copy) the required excises, then give solutions by *yourself*.

**Copy excises each other is NOT permitted!**

EX15

1. P456: 10.6. P457: 10.9, 10.10.

2\* Let  $X \sim N(0, I_n)$ ,  $A = A_1 + A_2$  where  $A_1, A_2$  are  $n \times n$  symmetric matrices and  $A_2 \geq 0$ .

(1). Prove that  $X^T A X \sim \chi^2(r) \iff A^2 = A, \text{tr}(A) = r$ .

(2). Assume  $X^T A X \sim \chi^2(r)$ ,  $X^T A_1 X \sim \chi^2(r_1)$ , prove  $X^T A_2 X \sim \chi^2(r_2)$  with  $r_2 = r - r_1$ ,  $X^T A_1 X$  and  $X^T A_2 X$  are independent.